Technical Notes

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Effect of Forward Acceleration on Aerodynamic Characteristics of Wings

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I. Introduction

DURING the last several decades, substantial growth has been made in unsteady lifting surface theories. Most of these, however, concern out-of-plane motions with few studies for nonuniform flight speeds.[1-4] These problems have become increasingly more important, because recent airplanes, such as STOL or supersonic, can experience considerable acceleration during the takeoff climb or landing approach. More precise estimation of takeoff or landing distance would require some aerodynamic theories for accelerated flight.

Wagner,[1] Isaacs,[2] and Greenberg[3] presented theories for a lifting airfoil in nonuniform flight speeds in an incompressible inviscid flow. These three works, however, were developed for a two-dimensional thin airfoil. James[4] also presented a theory that was derived through the method of matched asymptotic expansions. His theory is applicable to three-dimensional wings having high aspect ratio. Since it is quite elaborate, its practical use seems to be limited to some special cases such as for short- and long-time limits, because of its mathematical complexity.

In the present Note, a new, simpler theoretical formulation is presented for a three-dimensional wing flying with nonuniform speed in an incompressible and inviscid fluid. In Sec. II, general formulations are presented, and in Sec. III, some applications for the lift problem are given. Many instructive physical insights are presented that have not been given by previous writers.

II. Basic Formulation

Throughout this paper, a system fixed on a moving body will be used which is called "moving axes system" in contrast to the usual "fixed (inertia) system." Expressions of the flow-tangency condition on body surfaces take forms similar to those for uniform flight speeds, by using the moving axes system. The x, y, and z axes are taken in the streamwise, spanwise, and normal directions, respectively, with the origin at the midchord of the midspan of the wing. Components of the disturbance velocity are u', v', and w'. The wing domain is denoted by S. Basic relations in terms of moving axes system are established in Ref. 5. Although these are written generally, we restrict ourselves to a case under the following assumptions: 1) the fluid is at rest except for the motion induced by the body; 2) the fluid is incompressible; 3) the wing lies nearly in the x-y plane

$$F_{L\pm} = z - f_{L\pm}(x,y,t) = 0 \tag{1}$$

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where ± concerns the upper and lower surfaces, respectively; and 4) the wing merely translates toward the negative x direction with a velocity $U(t)$, without in-plane rotations. Then the basic relations are written as follows:

$$\nabla^2\phi' = 0, \quad u' \equiv \text{grad}\,\phi' \tag{2}$$

$$p'/\rho = -[\partial/\partial t + U(t)\partial/\partial x]\phi' \tag{3}$$

$$w' = [\partial/\partial t + U(t)\partial/\partial x]f_L \tag{4}$$

The prime denoting disturbances will be omitted for simplicity hereinafter. Small quantities are f_L, ϕ, and w, whereas $U(t)$ is assumed to be arbitrary. Both f_L and $U(t)$ are prescribed quantities. Double signs ± are omitted in Eq. (4). Equations (2-4) include products—such as $U\phi$ and Uf_L—of two functions dependent on time. Thus, Fourier or Laplace transforms in time variables become useless. The concept of frequency response in time also is not applicable for the same reasons, but we can introduce it in the space variables (in-plane).

A. Flowfield Over Infinite Wavy Wall

Consider the semi-infinite space $z \geq 0$ over a wavy wall surface which lies close to the whole x-y plane. The wall may be described by the real part of a complex exponential function:

$$f_L(x,y,t) \equiv Re[\hat{f}_L(t;\alpha,\gamma)e^{i\alpha x + i\gamma y}], \quad i \equiv \sqrt{-1} \tag{5}$$

All other quantities may also be represented in a similar manner. Applying these to Eqs. (2-4), we obtain the disturbance pressure on the wall surface $z = +0$:

$$\frac{\hat{p}}{\rho} = R^{-1}\left[\frac{\partial}{\partial t} + i\alpha U(t)\right]\hat{w}(t), \quad R \equiv \sqrt{\alpha^2 + \gamma^2} \tag{6}$$

$$= R^{-1}\left[\frac{\partial^2}{\partial t^2} + 2iU\alpha\frac{\partial}{\partial t} - U^2\alpha^2 + i\alpha\dot{U}\right]\hat{f}_L \tag{7}$$

In the bracket of the right-hand side of Eq. (7), the first and second terms concern an unsteady out-of-plane motion, the third a steady motion, and the fourth an unsteady in-plane motion. These motions may be simply superimposed in the meaning of Eq. (7). Equations (6) and (7) are, as they stand, interpreted as the complex amplitude of the surface pressure over an infinite double-sinusoidal wavy wall, moving with a velocity $U(t)$. They, however, have greater utility, because Fourier superposition makes it possible to express the surface pressure over arbitrary finite wings.

B. Wing Having Symmetric Thickness Distribution

We can derive the well-known drag due to the virtual mass resulting from an in-plane acceleration. But the details should be omitted owing to the space limit.

C. Formulation for Lifting Wings

Consider the lifting surfaces. Both p and ϕ are odd in z, so only the flowfield in the semi-infinite space $z \geq 0$ need be investigated. Hence, we may start with the solution for a wavy wall. In contrast to the nonlifting case, however, the following cautions should be stated: 1) on the $z = 0$ plane, the upwash w is specified and p is unknown within S, while p is specified (zero) and w is unknown outside S; 2) treatment of

$\partial f_L/\partial t \neq 0$ is required, on starting with the solution for a wavy wall. Even if the wing geometry itself does not depend on time, the wake geometry may depend on time during nonuniform in-plane motions (even though observed from the moving axes system). This is the reason for caution 2). Thus, simple superpositions accepted for the nonlifting case become invalid. Regarding Eq. (6) as a first-order ordinary differential equation for $\hat{w}(t)$ and solving it, one obtains

$$\hat{w}(t) = \rho^{-1}\int_{t_s}^{t} \hat{p}(t_1,\alpha,\gamma)(\alpha^2+\gamma^2)^{1/2}\exp\{i\alpha[\tau(t_1)-\tau(t)]\}dt_1 \tag{8}$$

where t_s denotes the time at which the wing starts its motion. $\tau(t)$ is the distance covered by the wing motion.

$$\tau(t) = \int_{t_s}^{t} U(t_1)\,dt_1 \tag{9}$$

The convolution integral of Eq. (8) yields

$$w(x,y,t) = \frac{-1}{2\pi\rho}\int_{t_s}^{t} dt_1 \iint_S \frac{p(\xi,\eta,t_1)\,d\xi d\eta}{\{[x_0+\tau(t_1)-\tau(t)]^2+y_0^2\}^{3/2}} \tag{10}$$

For lifting problems the upwash velocity is affected by the full history of the wing motions—any explicit effect of an instantaneous acceleration could not be evaluated in contrast to nonlifting problems. Equation (10) may be useful to obtain the pressure distribution over the wing surface, when the normal wash is prescribed. Equation (10) agrees with Eq. (17) in Ref. 4.

III. Solutions for Lifting Cases

Equation (10) cannot be solved for general wing planforms and general types of upwash. However some considerations made up to now will be presented.

A. General Discussion

We restrict ourselves to the case where the velocity $U(t)$ is always nonzero positive. Then the time variable t may be replaced by the flight distance $\tau(t)$, because they correspond one-to-one with each other. Now Eq. (10) reads

$$\hat{w}(x,y,\tau) = \frac{1}{4\pi\rho}\iint_S d\xi d\eta \int_{\tau_s}^{\tau} \frac{\ell(\xi,\eta,\tau_1)\,d\tau_1}{[(x_0-\tau_0)^2+y_0^2]^{3/2}} \tag{11}$$

where

$$\ell(\xi,\eta,\tau) \equiv \Delta p(\xi,\eta,\tau)/U(t(\tau)) \tag{12}$$

$$\Delta p(\xi,\eta,\tau) \equiv p_-(\xi,\eta,\tau) - p_+(\xi,\eta,\tau) \tag{13}$$

$$\hat{w}(x,y,\tau) \equiv w(x,y,t(\tau)) \tag{14}$$

$$\tau_0 \equiv \tau-\tau_1, \quad x_0 \equiv x-\xi, \quad y_0 \equiv y-\eta \tag{15}$$

Hereinafter $\hat{w}(x,y,\tau)$ will be simply written as $w(x,y,\tau)$, without confusion. Equation (11) may be regarded as the integral equation for a wing flying with unit velocity. Moreover we may replace the lower limit τ_s by $-\infty$, when the condition $w=0$ holds for $-\infty<\tau_1<\tau_s$. In what follows we will select the more convenient value of this lower limit between zero and $-\infty$. It is noteworthy that Eq. (11) may be rewritten in the following forms:

$$w(x,y,\tau) = \frac{-1}{4\pi\rho}\iint_S d\xi d\eta \frac{1}{y_0}\frac{\partial}{\partial y_0}\int_{-\infty}^{x_0}\frac{\ell(\xi,\eta,\zeta+\tau-x_0)}{(\zeta^2+y_0^2)^{1/2}}d\zeta \tag{16}$$

or

$$w(x,y,\tau) = \frac{1}{4\pi\rho}\iint_S \ell(\xi,\eta,\tau)K_{III}(x_0,y_0)\,d\xi d\eta - \frac{1}{4\pi\rho}\iint_S d\xi d\eta\int_{-\infty}^{\tau}\frac{\partial \ell}{\partial \tau_1}(\xi,\eta,\tau_1)K_{III}(x_0-\tau_0,y_0)\,d\tau_1 \tag{17}$$

where

$$K_{III}(x,y) = y^{-2}[1+x(x^2+y^2)^{-1/2}] \tag{18}$$

$K_{III}(x,y)$ is just the kernel for the steady case. It is instructive to consider the physical meanings of Eq. (17). In the right-hand side, the first term denotes the quasisteady effect, and the second the unsteady correction.

Equation (16) shows a simple linear relation between w and ℓ as functions of τ. Therefore, we may use Fourier superposition.

$$\begin{matrix} w \\ \ell \end{matrix}(x,y,\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \begin{matrix} w^* \\ \ell^* \end{matrix}(x,y,\omega)e^{i\omega\tau}d\omega \tag{19}$$

Substitution of Eq. (19) into Eq. (16) yields

$$w^*(x,y,\omega) = \frac{1}{4\pi\rho}\iint_S \ell^*(\xi,\eta,\omega)\lim_{z\to 0}\frac{\partial^2}{\partial z^2} \times \int_{-\infty}^{x_0}\frac{e^{i\omega(\zeta-x_0)}}{(\zeta^2+y_0^2+z^2)^{1/2}}d\zeta d\xi d\eta \tag{20}$$

Equation (20) is just the well-known integral equation for a wing making an out-of-plane simple harmonic motion in time. The preceding reduction suggests the approach, shown in Table 1, to solving arbitrary problems for a wing executing an in-plane motion.

B. Solutions for Special Cases

We will try analytical approaches for some simple cases. As expected they are slender wings and two-dimensional airfoils.

1. *Slender Wings*

One simple way which reduces the integral equation for general lifting surfaces to that for slender wings is to use the approximation[6] $|y_0| \ll |x_0|$ in the kernel functions. Then, Eq. (17) reads

$$w(x,y,\tau) = \frac{1}{2\pi\rho}\oint \frac{d\eta}{y_0^2}\int_0^x \ell(\xi,\eta,\tau-x+\xi)\,d\xi \tag{21}$$

Through Eq. (21) we can easily reproduce Eq. (34) in Ref. 7, for lift, given by

$$(L-L_{qs})/L_{qs} = (n/3)A_g, \quad A_g \equiv C_0 g/U^2,$$

C_0 = maximum chord length

where n, g, and L_{qs} denote $\dot{U}/g$, the acceleration of gravity, and the quasisteady lift, respectively. Let us show a numerical example. Assume $C_0 = 50$ m, $U = 100$ kts., and $n = 1$. Then we have $(L-L_{qs})/L_{qs} = 0.062$. Although this figure is not so large, it may be significant and should not always be negligible.

2. *Two-Dimensional Airfoil*

For the two-dimensional case, we use the Fourier superposition. Following the process shown in Table 1, we have the lift distribution for the problems where a wing flies with a nonuniform velocity $U(t)$.

Table 1 Approach for solving the integral equation of lifting wings—superposition due to Fourier integral

$\ell^*(\xi,\eta,\omega)$;	Solution of Eq. (20):	Flight velocity is unity, out-of-plane motion, simple harmonic in time
$\ell(\xi,\eta,\tau)$;	Through use of Eq. (19):	Flight velocity is unity, w is an arbitrary function in time (or τ)
$\Delta p=\ell\cdot U(t)$;	Through use of Eq. (12):	For variable (nonzero positive) flight velocity

$$\Delta p=\ell\cdot U(t)\equiv\Delta p_1+\Delta p_2+\Delta p_3 \tag{22}$$

The terms on the right hand side of Eq. (22) may be called "quasi-steady term," "instantaneous acceleration term," and "wake term," respectively. These terms are expressed as follows:

$$\frac{\Delta p_1}{\rho U}=\frac{2}{\pi}\int_{-1}^{1}\sqrt{\frac{1-\bar{x}}{1+\bar{x}}}\sqrt{\frac{1+\xi}{1-\xi}}\,\frac{w(\xi,\bar{\tau})}{\bar{x}-\xi}\,d\xi,\quad \bar{x},\bar{\tau}\equiv x,\tau\Big/\left(\frac{C_0}{2}\right) \tag{23}$$

$$\frac{\Delta p_2}{\rho U}=\frac{1}{\pi}\int_{-1}^{1}\frac{dw}{d\bar{\tau}}(\xi,\bar{\tau})\,\ell n\frac{(\bar{x}-\xi)^2+(\sqrt{1-\bar{x}^2}-\sqrt{1-\xi^2})^2}{(\bar{x}-\xi)^2+(\sqrt{1-\bar{x}^2}+\sqrt{1-\xi^2})^2}\,d\xi \tag{24}$$

and

$$\frac{\Delta p_3}{\rho U}=-\frac{2}{\pi}\sqrt{\frac{1-\bar{x}}{1+\bar{x}}}\int_{-1}^{1}\sqrt{\frac{1+\xi}{1-\xi}}\,d\xi\int_{\bar{\tau}_s}^{\bar{\tau}}\frac{dw}{d\bar{\tau}}(\xi,\bar{\tau})\times[\Phi(\bar{\tau}-\bar{\tau}_1)-1(\bar{\tau}-\bar{\tau}_1)]\,d\bar{\tau}_1 \tag{25}$$

where $\Phi(\bar{\tau})$ denotes the so-called Wagner function, and $1(\bar{\tau})$ the unit step function. In order to gain physical insight, let us consider a simple case where the airfoil is a flat plate having an incidence α. Then the lift component L_2 resulting from Δp_2 reads

$$L_2/L_{qs}=(n/4)A_g \tag{26}$$

This is positive, when n or acceleration is positive; this is a situation similar to that for a slender delta wing. L_{qs} is the lift component resulting from Δp_1. It is interesting to compare the present result with one obtained by James,[4] which is

$$\ell(s,\tau;A)=\pi\rho\dot{U}(C^2/A^2),\quad \text{for } t\ll 1, \text{ and } A\gg 1 \tag{27}$$

where α seems to be missing. [See Ref. 4 for symbols in Eq. (27)]. Hence, James' limiting lift is nothing but the present L_2. The last lift component L_3 resulting from Δp_3 is also easily calculated when the Wagner function $\Phi(\tau)$ is expressed

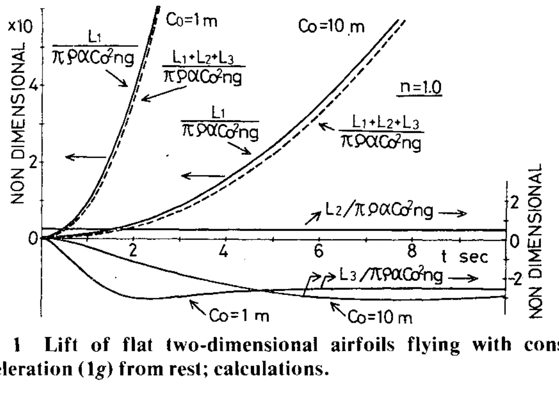

Fig. 1 Lift of flat two-dimensional airfoils flying with constant acceleration (1g) from rest; calculations.

by an appropriate approximate form. Since the explicit form of L_3 is somewhat lengthly and space is severely limited, only one figure will be presented. Figure 1 shows an illustrative numerical example, for $n=1$ and chord length $C_0=1$ and 10 m. The unsteady lifts (broken curves) are lower than the quasisteady lifts $L_1=L_{qs}$ (solid curves) except for some short initial periods, where L_2 predominates. It is noteworthy that the unsteadiness corrections are for the most part negative for two-dimensional airfoils, while they are positive for slender wings, for positive accelerations.

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Explicit Expression for the Smooth Wall Velocity Distribution in a Turbulent Boundary Layer

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Nomenclature

p	= static pressure
u	= mean velocity in x direction
u'	= component of instantaneous fluctuating velocity in x direction
u_0	= wall friction velocity $=\sqrt{\tau_0/\rho}$
u_*	= dimensionless velocity $=u/u_0$
v	= mean velocity in y direction
v'	= component of instantaneous fluctuating velocity in y direction
w_k	= wake function
x	= Cartesian coordinate in longitudinal direction
y	= Cartesian coordinate perpendicular to wall
y_*	= dimensionless wall distance $=yu_0/\nu$
B	= constant in logarithmic region of mean velocity distribution
C	= constant of proportionality
U_∞	= freestream velocity
δ	= boundary-layer thickness
κ	= von Karman constant
μ	= molecular viscosity
ν	= molecular kinematic viscosity
ν_t	= eddy kinematic viscosity

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